

Measuring Brand Awareness in a Random Utility Model

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Abstract. Brand Awareness is recognized to be an important determinant in shaping the success of durables [14, 12], yet it is very difficult to be quantified. This is exactly the main goal of this paper: propose a suitable model where brand awareness of two competing firms is modelled and, eventually, estimated. To this aim, we build a random utility model for a duopoly where each competitor is characterized by different pricing strategies and brand awareness. As a result, different levels of market shares will emerge at the equilibrium. As a case study, we calibrate the model with real data from the smartphone industry obtaining an estimate of the value of the brand awareness of two leading brands.

Keywords: Random utility model · social interactions · binary choice model · discrete choice model · duopoly · brand awareness.

1 Introduction

Brand awareness plays an important role in consumer decision making and is central on determining the success of companies. The objective of branding decisions on modern organizations is to generate a brand image for their products or services that is in line with the firms market target and positioning decisions. This will result in a strong association of consumers with the qualities and the image of a particular brand, a relation called brand awareness [6]. Joseph [7] and Keller [9] illustrate how brand's image can be present in the mind of consumers and how it impacts their buying decisions. O'Cass and Siahtiri [10] argue that consumption behaviors characterize the desire to possess certain brands as a mean to achieve status and self-fulfillment; therefore, brand awareness plays an important role in order to project a certain image on potential customers. For example, consumers buy a special cloths brand or car brand because they want to identify themselves with the brand and be associated with the brand image of prestige [8].

Branding and brand awareness also results in customers spreading brand loyalty and devotion: devoted consumers act to bring others attention to the brand and attract new customers. In addition, individuals are attracted towards the group of consumers with the highest recognition [4]. As argued by Asch [1],

“the primary mechanism in social influence is the change in the definition and meaning of an object”. Woods and Hayes [14] tell us how the consumers perform a “social (re)construction of reality”, reinterpreting the information about objects in relation to their reference groups, wondering whether this would be in line or not with their being, and thus leading to potential social reward or punishment. In Virvilaite et al. [13], a positive correlation was found between word of mouth, brand loyalty and brand awareness on luxury goods. Summarizing, it is evident how social interactions play an important role in decision making.

Our goal is to model brand awareness of two competitive firms in a society made of heterogeneous individuals with distinct preferences, where brand awareness plays a crucial role in the decision process. It produces a positive externality, increasing in the company position on the market. We micro found agents with heterogeneous preferences using a random utility model with a social component, in line with a traditional discrete social choice framework with social interactions. In their seminal paper [3], Brock and Durlauf propose a binary decision model where the action of single players is influenced by an aggregate signal represented by the (estimated) percentage of actors choosing one product or the other. In particular, actors are prone to imitate the behavior of the majority, thus, proving to be influenced by social interactions in their decision-making. In [11], the Brock and Durlauf paradigm is transferred into a game-theoretical language, thus, highlighting the strategic behaviour of a large population of players subject to social interactions.

In the present paper, we extend the model in [11] to the case of a duopoly: two competitors, characterized by different levels of brand awareness and prices, offer a new technology on the market. These quantities enter as parameters into a well-posed random utility model and, eventually, shape the level of market shares for the two technologies at the equilibrium. As an illustrative example, we calibrate the model using real data of the market share and the prices of the two major players in the smartphone industry: Apple and Samsung.

2 A Duopoly and a Large Population of Possible Buyers

Each agent in a large population of N possible buyers has to decide among three mutually exclusive options: buying product A , buying product B or stay out of the market. Each action yields a utility U_1 , U_2 and U_0 , respectively. The decision-making process can be visually represented by the decision tree depicted in Figure 1.¹

Starting from the left, the agent faces two subsequent decisions: (i) to buy or not to buy the product, (ii) which one between the two to buy. If $U_0 > \max(U_1, U_2)$, we end up in *Event 0*. Without loss of generality, we set $U_0 = 0$.

¹ Square decision nodes indicate the moment at which the agent is required to take a decision, while triangles nodes are terminal nodes endowed with a utility value that the agent receives if the node is selected. The share of the population that decides to enter the market is s , while x is the share of consumers in the market that purchase good A .

On the other hand, as soon as the utility of buying product A or B is positive, the agent follows the higher branch and chooses the preferable outcome according to utilities U_1 and U_2 .

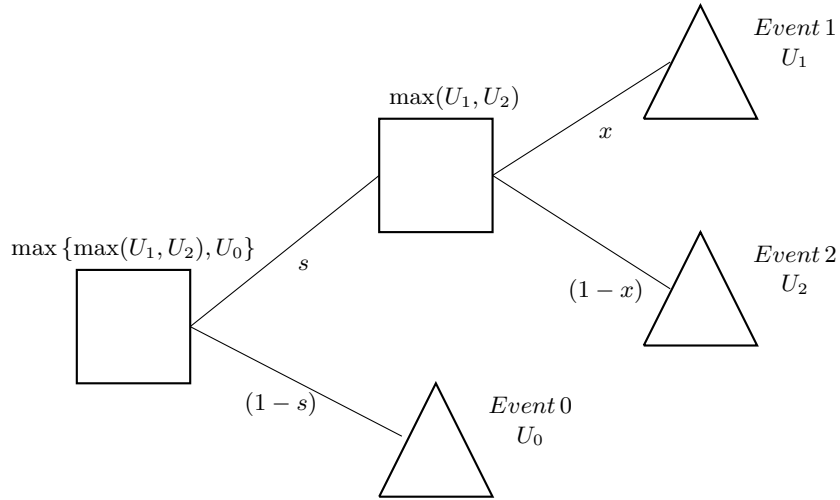


Fig. 1. Decision Tree representing to the decision process of the potential adopters.

Let us define P_A and P_B as the probabilities of buying A or B , s the total probability of entering the market and x as the probability of *Event A* conditioned on the fact that the agent enters the market. In formulae

$$s = P_A + P_B = P(\max(U_1, U_2) > 0), \quad (1)$$

$$x = \frac{P_A}{P_A + P_B} = P(U_1 > U_2 | \max(U_1, U_2) > 0). \quad (2)$$

In order to compute the values of s and x at the equilibrium, we rely on a random utility model inspired by [3, 11]. In details, for each single actor $i = 1, \dots, N$, we set

$$U_1(i) = -p_1 + q_1 x s + \epsilon_1(i), \quad (3)$$

$$U_2(i) = -p_2 + q_2 (1-x)s + \epsilon_2(i), \quad (4)$$

$$U_0(i) = 0. \quad (5)$$

(3) and (4) resemble the standard shape of random utilities à la Brock and Durlauf (see [3]) and are composed of three terms. p_1 and p_2 are the market prices of technology A and B respectively; each of the second components introduces an externality due to social interactions. Indeed, $x s$ and $(1-x)s$ denote

the respective market shares² prevailing at the equilibrium whereas q_1 and q_2 measure the level of brand awareness³; finally, $\epsilon_1(i)$ and $\epsilon_2(i)$ are *i.i.d.* random variables introducing heterogeneity in the population of buyers. Following standard literature in random utility models, we assume that $\epsilon_1(i)$ and $\epsilon_2(i)$ have a logistic probability distribution η with mean zero and variance $\sigma^2 = \frac{\pi^2}{3} = \frac{1}{\beta}$.

$$\eta(z) = P(\epsilon \leq z) = \frac{1}{1 - \exp(-\beta z)}, \quad \beta > 0. \quad (6)$$

The bigger σ^2 (the smaller β) the more disperse the taste of the population of buyers.

Each agent compares among his utilities of adopting product A , product B , and not entering the market, $\{U_1(i), U_1(i), U_0\}$, as it was described in Figure 1. The agent's utilities when entering the market, $\{U_1(i), U_2(i)\}$, depend on the action of other agents through the participation shares shaping the social component term of the utilities. This eventually results in setting a non-cooperative game, in which each agent makes his choice given an expectation of the population outcome. Similarly to [11], agents do not communicate or coordinate, rather each individual knows the common distribution of the heterogeneous shocks ϵ_j , for $j \neq i$. In other words, we impose *rational expectations*: each agent has a correct belief about others preferences; moreover, we assume that each agent shares the same expectation about other player's actions. For a fixed population of N agents, where $N \rightarrow \infty$, at least one Nash equilibrium in pure strategies exists (see [5]). The result of this game theoretical setting -when we let $N \rightarrow \infty$ - is therefore the emergence of a Nash equilibrium characterized by levels x^* and s^* . Because the logistic distribution is unimodal and S-shaped, we can have one or more equilibria depending on the set of prices $\{p_1, p_2\}$ and brand awareness $\{q_1, q_2\}$ (see [3]).

Under these assumptions, when the number of buyers tends to infinity, it is possible to derive an explicit expression for the probabilities P_A and P_B .⁴ All these results are summarised in the next proposition:

Proposition 1 *Assume a population of N agents as described by equations (3)-(5) where $\{\beta, p_1, p_2, q_1, q_2\}$, are fixed and where η_1 and η_2 have the form (6). Then, at least one Nash equilibrium (x_N^*, s_N^*) exists. Moreover, when $N \rightarrow \infty$,*

$$(x_N^*, s_N^*) \rightarrow (x^*, s^*)$$

² The number of agents, N , can be partitioned into $N = N^1 + N^2 + N^0$. Consequently the market shares can be defined as $1 = \frac{N^1}{N} + \frac{N^2}{N} + \frac{N^0}{N}$. Thus, when $N \rightarrow \infty$, $\lim_{N \rightarrow \infty} \frac{N^0}{N} = 1 - s$, $\lim_{N \rightarrow \infty} \frac{N^1}{N} = sx$ and $\lim_{N \rightarrow \infty} \frac{N^2}{N} = (1 - x)s$.

³ The coefficient multiplying the social component of the utility is interpreted as the force of externality or as the imitation driver (see for example [2], where a similar interpretation applies to the context of diffusion of innovation). In the same spirit, we interpret it here as the brand awareness of the issuing firm.

⁴ For a derivation, see the Appendix

where (x^*, s^*) solves the fixed point problem

$$\begin{cases} f(x, s) = 0 \\ g(x, s) = 0 \end{cases} \quad (7)$$

with $f(x, s) := P_A + P_B - s$ and $g(x, s) = \frac{P_A}{P_A + P_B} - x$. Moreover,

$$\begin{aligned} P_A = & \frac{\exp(\beta X_0)}{(\exp(\beta X_0) - 1)^2} \cdot \log \left(\frac{\exp(-\beta X_0) + \exp(\beta X_1)}{\exp(\beta X_1) + 1} \right) \\ & + \frac{\exp(\beta X_0)}{(\exp(\beta X_0) - 1)(\exp(\beta X_1) + 1)} \end{aligned} \quad (8)$$

and

$$\begin{aligned} P_B = & \frac{-\exp(\beta X_0)}{(\exp(\beta X_0) - 1)^2} \cdot \log \left(\frac{\exp(\beta X_2) + 1}{\exp(\beta X_0) + \exp(\beta X_2)} \right) \\ & - \frac{1}{(\exp(\beta X_0) - 1)(\exp(\beta X_2) + 1)} \end{aligned} \quad (9)$$

where $X_1 = p_1 - q_1 x s$, $X_2 = p_2 - q_2(1 - x)s$ and $X_0 = p_2 - p_1 - q_2 s + (q_1 + q_2) s x$.

Note that the problem is intrinsically bi-dimensional in that x and s have to be determined at the equilibrium as the solutions to (7). Depending on the values of the parameters $(\beta, q_1, q_2, p_1, p_2)$, different equilibria emerge. Indeed, all the equilibrium solutions (x^*, s^*) can be found by solving

$$(x^*, s^*) = \underset{(x, s) \in [0, 1] \times [0, 1]}{\operatorname{argmin}} \{ \phi(x, s) \}, \quad (10)$$

where

$$\phi(x, s) = f(x, s)^2 + g(x, s)^2. \quad (11)$$

As an example, we run a simulation where we consider a fixed population (represented by $\beta = 2$). In Figure 2 we plot the contour levels of equation (11), when firm 1 has a stronger brand awareness ($q_1 = 4, q_2 = 1$), but the price of firm 2 is more competitive ($p_1 = 1.5, p_2 = 1$). The black dots represent the solution points (x^*, s^*) of (10) depicted at points $(0.35, 0.23)$, $(0.66, 0.38)$ and $(0.99, 0.99)$. The top left corner displays the equilibrium point $(0.99, 0.99)$, which illustrates the strong effect of brand awareness and social interaction, where Firm 2 is practically taken out of market. Moreover, even at the most favourable equilibrium point for Firm 2, $(0.35, 0.23)$, Firm 1 still possesses an important market share and still competes on the market, although the total market share (s) is considerably reduced.

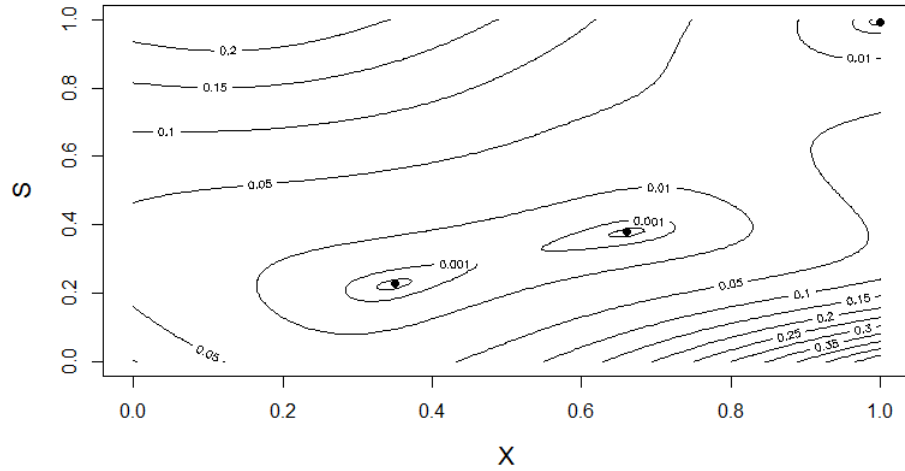


Fig. 2. Contour levels and equilibrium points (x^*, s^*) of the function $\phi(x, s)$

In Table 1 we collect the values of (x^*, s^*) emerging at the equilibrium for different specifications of the parameters. The first row shows a baseline scenario where the two firms share equal characteristics and where three equilibria emerge⁵. When p_1 increases (second row), there is only one equilibrium, at which the market share of product B is dominant, when q_1 increases (third row) the market share of product A is dominant. The fourth row shows how the negative effect of p_1 can be attenuated when q_1 is high. Additionally, we can see that when brand devotion is high, the total market size s increases. The fifth row shows a scenario similar to the first one, where both products share equal characteristics; however, the prices are twice higher, yet brand awareness is cut by half. In this case the positive effect of brand awareness is not strong enough to offset the negative effect of higher prices. Consequently, there is only one equilibrium where firms share equal market share but where the total market size is considerably reduced. Finally, the last row shows exactly the scenario depicted on Figure 2, which is similar to row 4, except that p_2 is higher and q_2 is lower. As a result, both the medium equilibrium and the one where product B is dominant show a smaller total market size, as well a stronger market share for firm A .

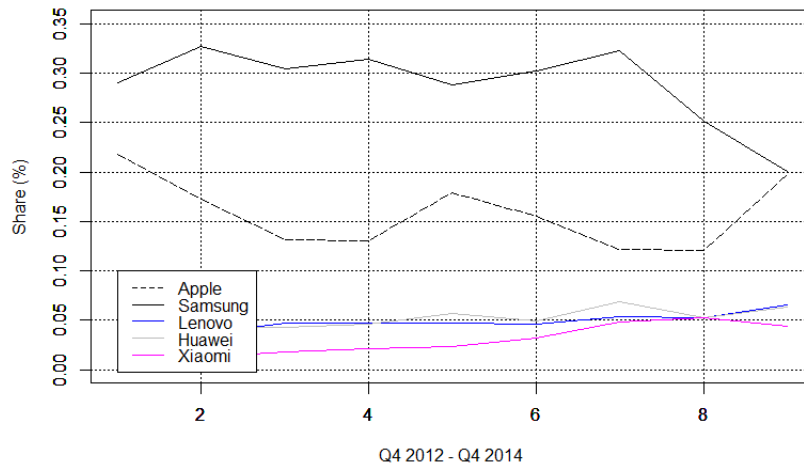
⁵ The presence of multiple equilibria is due to the non-linearity of the system (7) and has significant consequences on the strategic behavior of firms (see, for instance, [11]). In standard random utility models, it is shown that for q large enough, multiple equilibria may appear. In our setting, the picture is less clear because of the presence of the two competitors. For the sake of brevity, we leave this issue to future further investigation.

Table 1. Equilibrium values of (x, s) for different values of the parameters.

Parameters				Equilibrium Points
q_1	q_2	p_1	p_2	(x^*, s^*)
2	2	0.5	0.5	(0.90, 0.95), (0.50, 0.91), (0.10, 0.95)
2	2	1	0.5	(0.04, 0.94)
4	2	0.5	0.5	(1.00, 1.00)
4	2	1.5	0.5	(0.99, 0.99), (0.54, 0.9), (0.01, 0.94)
1	1	1	1	(0.50, 0.28)
4	1	1.5	1	(0.99, 0.99), (0.66, 0.38), (0.35, 0.23)

3 A Case Study: The Smartphone Industry

We apply our model to the case of the smartphone industry. We used the Gartner iDC data⁶ and extract quarterly market shares (Q4 2012-Q4 2014), illustrated on the following figure:

**Fig. 3.** Smartphone manufacturer industry market share, taken from Gartner iDC.

Looking at Figure 3, representing the market shares of the major competitors in the phone industry, it is easy to spot the predominance of the Samsung-Apple duo, holding under their control 40% of the global market, with their best performing competitors lagging more than 13 points behind. For this reason and for the importance that socio-psychological dynamics assumed in the choice of our next smartphone, we believe that the smartphone industry represents a

⁶ Data taken from Gartner, iDC at <http://www.gartner.com>

perfect case study to test the model being presented.

Samsung and Apple have adopted different strategies when it comes to their offered portfolio of products. Apple offers a small variety of products compared to a considerably wider selection offered by Samsung. In our analysis, we focus on the S5 and S6 Samsungs smartphones, and the iPhone 6 family plus the iPhone 5S Apples collection.⁷ The average price⁸ for a Samsung model is €550, while for Apple is €729. We let Apple represent product A and Samsung product B . Being the model of comparative nature, we normalize $p_2 = 1$ and $q_2 = 2$.⁹ Then, we set $p_1 = \frac{729}{550} = 1.325$. Finally, q_1 and β will be calibrated. To this aim, we rely on equation (11), taking s and x as suggested from the quarterly market shares (Q4 2012-Q4 2014) of the smartphone industry. Figure 4 shows the estimated brand awareness ratio q_1^*/q_2 , for the different quarters under analysis.

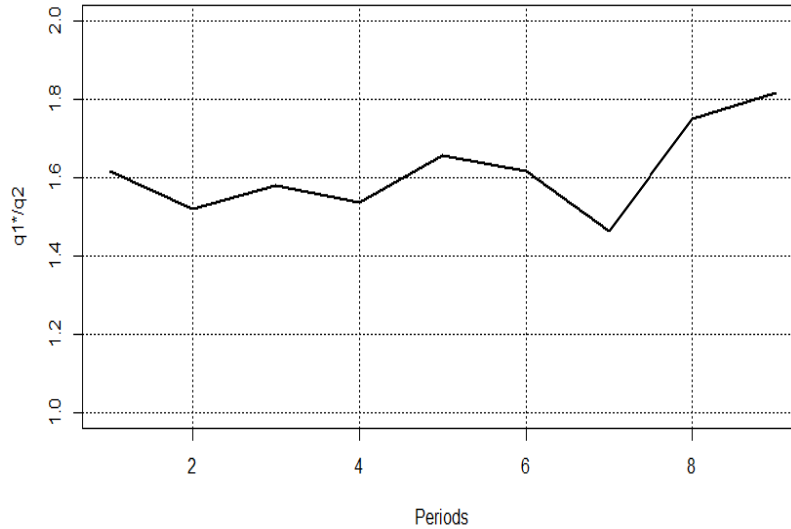


Fig. 4. Estimated brand awareness q_1^*/q_2 , when $q_2 = 2$.

⁷ Taking into account the products release date and Apples higher prices, the Samsung S5 and S6 in all of its different version represent the main competitors of Apples products.

⁸ Samsung: S6 Edge Plus (839), S6 Edge (€739), S6 (€739), S5 Neo (€330), S5 (€410), S5 Mini (€240). Apple: iPhone 6S Plus (€779), iPhone 6S (€889), iPhone 6 Plus (€779), iPhone 6 (€669), iPhone 5S (€529).

⁹ The value of $q_2 = 2$ has been chosen after a careful preprocessing of the model. We have tested it using q_2 set equal to $\{1, 2, 3, 4\}$. When $q_2 = 1$, q_1^* results to be negative (not feasible); while when $q_2 > 2$, the value of β^* falls between 5 to 50 (too high).

Our model shows a consistent higher level of Apple brand awareness compared to Samsung. It is no hyperbole stating that Apple consumers make religious references to Steve Jobs and Apples products, to the point that some of them are used to queue for days waiting for the release of the latest iPhone. Our model thus corroborates the well-known fact that the brand awareness of Apple is higher than the one of other firms. The novelty brought about by this model consists in its capacity to provide estimates of the ratio between the two main competitors brand awareness; moreover, the values we find are consistent with price and market shares data. The maximum points of the ratio of Apples to Samsungs awareness degree corresponds to the quarters in which Apple introduced its new products on the market: i.e., period 5 (Q4-2013) and 9 (Q4-2014). The decline after the fifth period could reflect the disappointment of the consumers who would have preferred to see a completely new smartphone, instead of an updated version of the old one (i.e., the iPhone 5S Apple release). Such a result shows that no matter how strong Apples brand is, there is still a chance for other brands to capture a slice of its market share. Finally, our calibration exercise also provides an estimate $\sigma^2 = 0.55^{10}$ for the variance of the logistic distribution characterizing the spread of taste among the buyers. This figure, often used in random utility models, is rarely calibrated to real data.

4 Final Remarks

Social interaction plays an important role in order to determine the success of goods or services, specially when consumers react differently to brand images that firms promote to capture their attention [1, 13, 14]. In this paper we have modelled consumer decision making under the assumption that brand awareness plays the role of a social component in the utility. We suggest a different way to look at and use random utility models. A large number of agents faces two subsequent choices: adopt or not a new technology and, eventually, which one between the two releases of the technology to buy. Dealing with a duopoly, a peculiarity of our model is the presence of two bunches of parameters characterizing the two competitors. Once provided the equations needed to determine the market shares at the equilibrium, we calibrate the model with real data related to the major players on the smartphones industry, obtaining a quantitative measure of the brand awareness ratio of the two competitors. Our results support the fact that Apple's brand awareness of Apple is higher than the one of other firms.

¹⁰ The average estimated β^* across the evaluated periods is 1.81.

A Appendix

Proof of Proposition 1

The existence of Nash equilibria and the convergence of (x_N^*, s_N^*) to the points solving (7) can be deduced from arguments developed in [5]. We now concentrate on the shape of the functions $f(x, s)$ and $g(x, s)$, hence on P_A and P_B . From (3) and (4), it follows that

$$P(U_1(i) > 0) = P(-p_1 + q_1xs + \epsilon_1(i) > 0) = P(\epsilon_1(i) > X_1) \quad (12)$$

$$P(U_2(i) > 0) = P(-p_2 + q_2(1-x)s + \epsilon_2(i) > 0) = P(\epsilon_2(i) > X_2) \quad (13)$$

where $X_1 = p_1 - q_1xs$, $X_2 = p_2 - q_2(1-x)s$ and $X_0 = p_2 - p_1 - q_2s + (q_1 + q_2)xs$. Therefore, (8) can be derived as follows:

$$\begin{aligned} P_A &= P(U_1(i) > U_2(i), U_1(i) > 0) = P(\epsilon_2(i) - \epsilon_1(i) < X_0, \epsilon_1(i) > X_1) \\ &= P(\epsilon_2(i) < X_0 + \epsilon_1(i), \epsilon_1(i) > X_1) \\ &= \int_{X_1}^{\infty} \eta(X_0 + \xi) d\eta(\xi) \\ &= \int_{X_1}^{\infty} \left(\frac{1}{1 + \exp(-\beta(X_0 + \xi))} \right) \frac{\beta \exp(-\beta\xi)}{(1 + \beta \exp(-\beta\xi))^2} d\xi \\ &= \int_{X_1}^{\infty} \frac{\beta}{(\exp(-\beta\xi) + \exp(-\beta X_0))(1 + \exp(-\beta\xi))^2} d\xi \\ &= \frac{\exp(\beta X_0)}{(\exp(\beta X_0) - 1)^2} \cdot \log \left(\frac{\exp(-\beta X_0) + \exp(\beta X_1)}{\exp(\beta X_1) + 1} \right) \\ &\quad + \frac{\exp(\beta X_0)}{(\exp(\beta X_0) - 1)(\exp(\beta X_1) + 1)}. \end{aligned}$$

We used the convolution formula for independent random variables to derive the third line and the form of the logistic distributions of η_1 and η_2 to derive the fourth. The latter expression follows by direct integration: differently from the normal random variable, an explicit expression for the logistic distribution can be provided.

Similarly, (9) is derived as follows:

$$\begin{aligned}
P(U_2(i) > U_1(i), U_2(i) > 0) &= P(\epsilon_2(i) - \epsilon_1(i) > X_0, \epsilon_2(i) > X_2) \\
&= P(\epsilon_1(i) < \epsilon_2 - X_0(i), \epsilon_2(i) > X_2) \\
&= \int_{X_2}^{\infty} \eta(\xi - X_0) d\eta(\xi) \\
&= \int_{X_2}^{\infty} \left(\frac{1}{1 + \exp(-\beta(\xi - X_0))} \right) \frac{\beta \exp(-\beta\xi)}{(1 + \beta \exp(-\beta\xi))^2} d\xi \\
&= \int_{X_2}^{\infty} \frac{\beta}{(\exp(\beta\xi) + \exp(\beta X_0))(1 + \exp(-\beta\xi))^2} d\xi \\
&= \frac{-\exp(\beta X_0)}{(\exp(\beta X_0) - 1)^2} \cdot \log \left(\frac{\exp(\beta X_2) + 1}{\exp(\beta X_0) + \exp(\beta X_2)} \right) \\
&\quad - \frac{1}{(\exp(\beta X_0) - 1)(\exp(\beta X_2) + 1)}.
\end{aligned}$$

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