Network Effects on Complex Adaptive Systems Approach to Modeling Human and Nature Dynamics

Patrick deWerk Neal[1], Zining Yang[1,2], Mark Abdollahian[1]

[1] Claremont Graduate University, Claremont, CA, 91711, USA
[2] La Sierra University, Riverside, CA, 92505, USA
{zining.yang, mark.abdollahian}@cgu.edu, patrick.d.neal@gmail.com

Abstract. In the context of sustainable development, complex adaptive systems frameworks can help address the coupling of macro social, environmental and economic constraint and opportunity with individual agency. Using a simple evolutionary game approach, we fuse endogenously derived socio-economic system dynamics from human and nature dynamics (HANDY) theory with Prisoner's Dilemma spatial intra-societal economic transactions. We then explore the potential of spectral information from the social network adjacency matrices to predict synchronization dynamics.

Keywords: Sustainable development · Complex adaptive systems · Network Analysis· Agent-based modeling · System dynamics · Game theory

1 Introduction

Social scientists have long identified dynamic linkages between economic development, population dynamics, and environment [8][6][11]. Starting in ecological economics, the human and nature dynamics (HANDY) perspective is a quantitative, trans-disciplinary approach to understanding modernization and development through interdependent economic and social forces at the aggregate society level. Here we extend previous work by Motesharrei.'s [11] novel systems dynamic representation of the theory at the societal level towards integrated macro-micro scales in a complex adaptive systems framework using an agent based approach. As macroscopic structures emerging from microscopic events lead to entrainment and modification of both, co-evolutionary processes are created over time [13]. Similar to Abdollahian et al [1-3] and Yang [15], we posit a new, approach where agency matters: individual game interactions, strategy decisions and outcome histories determine an individual's experience. These decisions are constrained or incentivized by the changing macro economic, cultural, social and political environment via human and nature dynamics theory, conditioned on individual attributes at any particular time. Emergent behavior results from individuals' current feasible choice set, conditioned upon past behavior and macro outcomes. Conversely, progress on economic development, the formation of cultural mores, societal norms and democratic preferences emerge from individuals' behavior interactions.

To explore potential real-world applications of this analysis, we consider the potential explanatory power of information contained in the eigenspectrum of the Laplacian matrix describing the dynamic adjacency matrices of the underlying social network of relationships between competing agents. As our extension of the HANDY model takes place, like so many complex processes, in a network setting, the time to equilibrium, or the time to more weakly defined states such partial synchronization or effective cooperation, is a quantity of key interest. It has been demonstrated in [4] that spectral information can anticipate topological scales, and in certain oscillator models this allows anticipation of the time to synchronization. Other work, such as [13], has extended this research into less stylized problems, including models with non-monotonic and non-linear paths to synchronization, with some success. We follow in this vein. This approach, borrowed from the theoretical physics literature, allows potential mean-field style analysis of an otherwise intractably complex game, and potentially the ability to anticipate the outcome and temporal characteristics of complex games featuring artificial intelligence without the need for complete and exhaustive simulation.

2 HANDY Background

HANDY postulates a development process in which inequality and use of resources play a critical role. Brander and Taylor [5] developed an ancestor model of population and renewable resource dynamics and demonstrated that reasonable parameter values can produce cyclical feast and famine patterns of population and resources. Their model shows that a system with a slow-growing resource base will exhibit overshooting and collapse, whereas a more rapidly growing resource base will produce an adjustment of population and resources toward equilibrium values. However, this approach does not include a central component of population dynamics: economic stratification and the accumulation of wealth.

Inspired by a Lotka-Voltera model at the core, Motesharrei et al. [11] develop a human population dynamics model by adding accumulated wealth and economic inequality. They develop and measure "carrying capacity" and show it to be a potentially practical means for early detection of collapse. When a population surpasses the carrying capacity, starvation or migration can threaten to significantly impact population levels and rates of change. However, humans can also accumulate wealth and then draw down resources when production cannot match consumption needs. Empirically, they posit that accumulated surpluses are not evenly distributed throughout society. As elites control resources normally, they could leave the mass of the population, while producing a portion of generated wealth, with only a small portion of it usually at or just above subsistence levels [7][4]. While the Brander–Taylor model has only two equations, Motesharrei et al's model adds an additional two equations to predict the evolution of nature, accumulated wealth, elites and commoners as an interdependent, asymmetric first order system. Their HANDY equations are given by:

$$\begin{cases} \dot{x}_C = \beta_C x_C - \alpha_C x_C \\ \dot{x}_E = \beta_E x_E - \alpha_E x_E \\ \dot{y} = \gamma y(\lambda - y) - \delta x_C y \\ \dot{w} = \delta x_C y - C_C - C_E. \end{cases}$$

In this system of equations, the total population is divided between the two variables, x_C and x_E, representing commoners and of elites respectively. The population grows at a birth rate β and decreases at a death rate α. In their model, β is assumed to be constant for both elites and commoners but α depends on wealth. The equation for nature includes a regeneration or gain term $\gamma Y(\lambda - Y)$, and a depletion or loss term $-\delta \chi_C Y$. Technological change can make the use of resources more efficient, but it also tends to raise both per capita resource consumption as well as resource extraction scales. Thus accumulated wealth increases with production, $\delta \chi_C Y$, and decreases with the consumption of the elites and the commoners

3 An Agent-Based, Complex Adaptive Systems Approach

While innovating a formal a systems approach for HANDY theory, a limitation of Motesharrei et al's [11] work lacks coupling and interdependence across human scales, from individuals to institutions and finally the societal outcomes they generate. Inspired by Motesharrei et al., our agent-based, complex adaptive systems HANDY model uniquely combines the interactive effects and feedbacks between individual human agency as well as the macro environmental constraints and opportunities that change over time for any given society. Decisions by individuals, including both elites and commoners, are affected by other individuals, social context, and system states, including accumulated wealth and resources. These decisions have variegated first and second order effects, given any particular system state or individual attributes. Such an approach attempts to increase both theoretical and empirical verisimilitude for some key elements of complexity processes, emergence, connectivity, interdependence and feedback [10] found across all scales of development.

We instantiate a non-cooperative, socio-economic Prisoner's Dilemma (PD) transaction game given agent i's attribute vector (A^i) of individual agent attributes similarity to agent j (A^j) for any A^{ij} pairs. The motivation behind this is that individuals are more likely to interact, engage and conduct transactions with other agents of similar norms [14] and produce different co-evolutionary behavior via frequency and rate dynamics [9]. To allow complexity, nonlinear and emergent behavior, we first randomly choose 50% of spatially proximal agents as sources who can choose a partner at each iteration t. The remaining targets are chosen by other agents based on symmetric preference rankings and asymmetric neighborhood proximity distributions. Following Abdollahian et al. [1-3] and Yang [38], we explore communications reach, social connectivity and technology diffusion that constrains the potential set of A^{ij} game pairs through talk-span.

We specifically model socio-economic transaction games as producing either positive or negative values to capture both upside gains or downside losses. Subsequently, A^{ij} games' V^{ij} outcomes condition agent W^i_{t+1} values, modeling realized costs or benefits from any particular interaction. The updated $W^i_{t+1} = W^i_t + A^{ij}$ game payoff for each agent subsequently gets added to the individual's attributes for the next iteration. We then repeat individual endogenous processing, aggregated up to society as a whole and repeat the game processes for $t+n$ iterations.

Aggregated wealth gets transformed into macro-society levels and impacts nature consistent with standard ecological economics as involving both inputs from, and outputs to nature, through depletion of natural sources and carrying capacity. The sum of all prior individual behavioral histories, evolutionary through iterations, does contribute to each individual and societal current states as an initial effort at a scale integrated framework. Thus agents simultaneously co-evolve as strategy pair outcomes at t to impact W^i at $t+1$, thus driving both positive and negative feedback process through $t+n$ iterations. These shape A^i attributes which spur adaptation to a changing environment. Feedback into subsequent A^{ij} game selection networks and strategy choice yields a complex adaptive system representation across multiple scales.

The resulting networks provide a rich dataset ripe for spectral analysis. By considering spectral gap metrics, the characteristic times between stable periods can be regressed against predictive qualities of the socio-economic transaction games networks. Differing slightly from Neal [12], specifically this analysis proceeds on the basis of using preselected values of the maximum eigenvalue gap max for a specific t, the average eigenvalue gap, and the median eigenvalue gap, measured after a 100 period burn-in without regard to the eventual end of the disorder period. This is as opposed to using the maximum value of any of these measures observed during the disorder period, as doing so would bias longer disorder n(t) periods towards higher maximum observations simply by means of more draws.

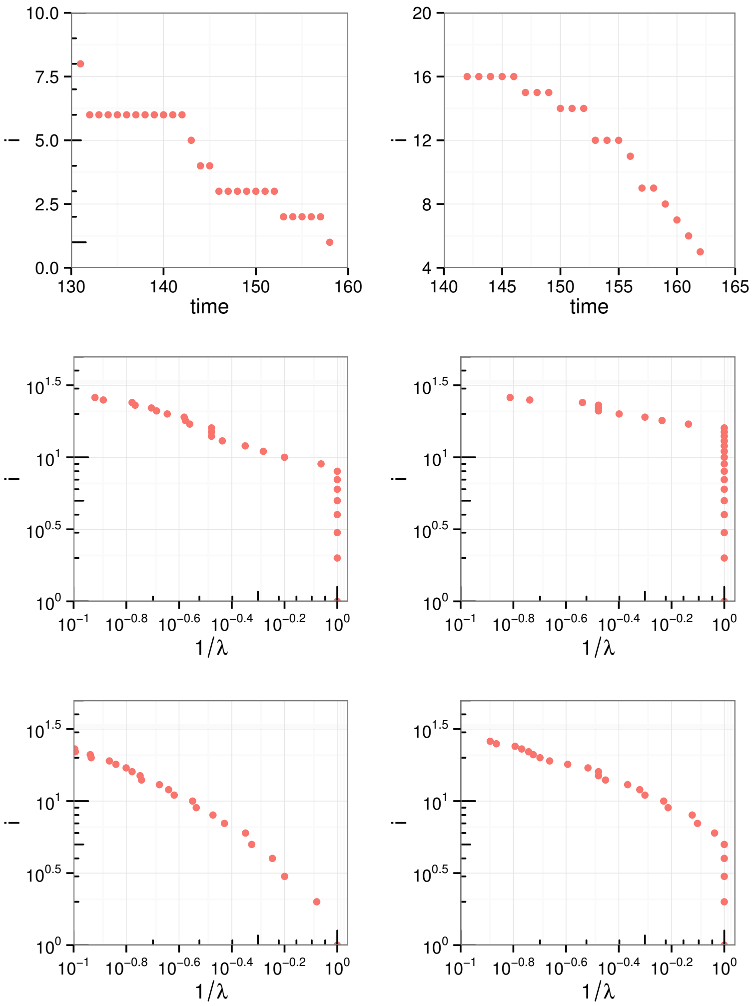

Fig. 1. Thresholds Comparasion

Above we detail a generic example of the type of information and results such analysis can provide, examining whether or not the eigenspectrum of the Laplacian matrix of the system under examination is able to predict the sequence of synchronization over the period of examination. Fig.1 [13] gives two such examples of thresholds which generate a monotonically decreasing number of separate components over time. The top row of Fig. 1 examines the time evolution Phase of synchronization of GDP for two networks, defined by thresholds of 0.976, 0.99, respectively. The second row displays the index of eigenvalues from the Laplacian, in ascending order, against the inverse of the eigenvalues themselves, from the Laplacian matrix derived from the adjacency matrix.

4 Methodology

Utilizing this agent-based version of HANDY, we specify the formation of an arbitrary (but substantial) level of cooperative links as the designator of a synchronized state in the model. This follows intuitive logic as well as the general incentive structure of the game underlying interactions in our extension of HANDY. Alternatively, the emergence of a large network of tributary relationships, akin to a stable and effective resource gathering hierarchy, could equally well be the expected, synchronized end-state given different payoff structures in the fundamental PD game.

Fig. 2 shows a sample time series charting the evolution of the number of cooperative links over the course of a particular simulation run. It is easily observed to be an increasing, but non-linear, process. The time occurring before this synchronization is achieved is denoted the *disorder period*.

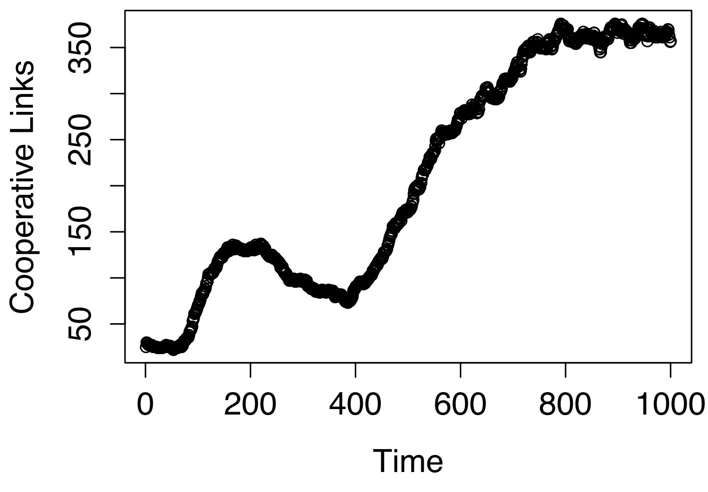

Fig. 2. Sample Time Series

Several agent-level variables offer a potential basis for the creation of the *dynamic connectivity matrix*. This is a realistic analogy to real life, where individual populations, corporations, species, etc, can be characterized by any of a variety of potential metrics. In this case, we have chosen to measure connectivity on the basis of societal wealth as captured in both the original HANDY model and our extension, as this variable figures heavily in the strategy selection process in our model.

As described above, a burn-in period is allowed to take place before variables are utilized. Then, pair-wise correlation statistics ϱ_{ij} are collected. Following Arenas et al [4], the *dynamic connectivity matrix* is constructed such that

$$\mathcal{D}_t(\tau)_{ij} = \begin{cases} 1 & \text{if } \rho_{ij}(t) > \tau \\ 0 & \text{if } \rho_{ij}(t) < \tau \end{cases}$$

From this adjacency matrix, the Laplacian matrix L is calculated such that individual entries are defined as $L_{ij} = k_i\delta_{ij} - a_{ij}$, where k_i is the degree of node i (that is, the number of edges it has in the network described by the adjacency matrix), δ_{ij} is the Kronecker operator (set to 1 for non-zero elements of the original adjacency matrix), and a_{ij} is the entry from the original adjacency matrix.

The penultimate step in data capture, for the purpose of moving on to a statistical analysis of the explanatory power of the eigenspectrum over population dynamics and synchronization in our extension of HANDY, is the calculation of the spectrum *S(L)* itself. The spectrum is traditionally presented in order from largest to smallest eigenvalue. From this vector, the gaps in the spectrum – and the maximum, mean, and median gaps – are recorded for every simulation from the single chosen Laplacian of the dynamic connectivity matrix.

5 Results

Holding instantiation parameters constant, we extract wealth figures and connectivity matrices over 300 runs of our extended model. Various spectral measures are tested for explanatory power with respect to the disorder period. One variable, the maximum gap between eigenvalues in the eigenspectrum of the Laplacian, is highly significant. However, overall predictive power of the disorder period in this model using spectral information alone is very low.

As seen in Table 1 below, the signs of the maximum and mean eigenspectrum gap coefficients are positive, while the median gap is negative. The significant coefficient for the maximum eigenspectrum gap can be interpreted as implying an additional 70 periods of disorder before cooperative synchronization for an increase of 1 in the largest gap between ordered eigenvalues. In the theoretical physics literature, similar relationships between spectral gaps and frustration time have been observed; in those works, this phenomenon is directly related to the distance, in degrees, between frustrated communities of oscillating elements; the larger the gap, the less likely the random (or, random-like) fluctuations of one element are to coincidentally match those of another community and reinforce the ensemble cycle. In the case of some parsimonious models, this process can be analytically explained by manipulation of the system's differential equations near equilibrium.

A similar process may be at work here, although the underlying behavior is much more complex than that of a simple oscillator, the implicit community structure is also likely to be more complex than the highly stylized hierarchies studied in that literature, and analytical solutions are all but infeasible. While the statistical significant seen here is encouraging, the explanatory power is essentially zero.

Table 1. Threshold 200 individual result.

	Dependent variable: Disorder Period		
	(1)	(2)	(3)
Max Eigen Gap	70.17*** (24.81)		
Mean Eigen Gap		906.45 (597.14)	
Median Eigen Gap			-732.62 (591.34)
Constant	206.49*** (34.48)	157.66* (93.22)	302.87*** (12.66)
Observations	300	300	300
R^2	0.03	0.01	0.01
Adjusted R^2	0.02	0.004	0.002

Note: * p<0.1; ** p<0.05; *** p<0.01.

Table 2, below, details the results of utilizing different combinations of the mean and median spectral gap measures with the maximum gap measure. The maximum gap metric remains significant, the other variables remain insignificant, and the R^2s remain at essentially zero. Results from additional experiments utilizing spectral information from connectivity matrices created earlier in simulation runs contained no statistically significant information and have been omitted.

Table 2. Threshold 200 multivariate result.

	Dependent variable: Disorder Period		
	(1)	(2)	(3)
Max Eigen Gap	67.00*** (28.09)	66.61*** (25.35)	61.07** (29.17)
Mean Eigen Gap	162.05 (669.66)		263.63 (683.34)
Median Eigen Gap		-417.85 (597.69)	-463.90 (610.34)
Constant	185.54** (93.23)	213.91*** (36.10)	180.63* (93.52)
Observations	300	300	300
R^2	0.03	0.03	0.03
Adjusted R^2	0.02	0.02	0.02

Note: * p<0.1; ** p<0.05; *** p<0.01.

Table 3 features analysis of data generated under slightly different circumstances. Simulations in this case were run until the relationships between artificial societies grew to 300 cooperative links; this slightly stronger selection of a synchronization threshold means longer run periods, generally. (See Fig. 3, below.) As such, we study information from dynamic connectivity matrices calculated at period 100 as well as 400. Spectral information from the earlier connectivity matrix is essentially null of statistical significance and is not displayed. The resulting spectral information from the connectivity matrix at period 400 is similar to that show in Table 1; statistically significant in some cases, but void of explanatory power.

Table 3. Threshold 300 individual result.

	Dependent variable:		
	Disorder Period		
	(1)	(2)	(3)
Max Eigen Gap	168.85***		
	(32.90)		
Mean Eigen Gap		1917.68***	
		(431.65)	
Median Eigen Gap			-394.75
			(415.14)
Constant	567.77**	484.77***	728.27***
	(31.77)	(54.71)	(9.74)
Observations	300	300	300
R^2	0.08	0.06	0.003
Adjusted R^2	0.08	0.06	-0.0003

Note: * p<0.1; ** p<0.05; *** p<0.01.

We then extend our regression model to incorporate a current trend and level component of the eventual dependent variable (number of cooperative connections) at t = 400, in order to increase explanatory power and study potential uplift from the spectral measures.

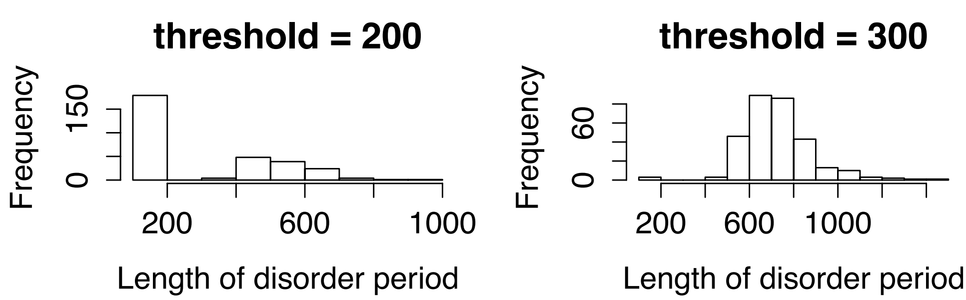

Fig. 3. Threshold 200 and 300 comparison.

The results can be seen in Table 4. Including the level and trend of cooperative connections alone lends an R^2 of 11%; however, including different combinations of the spectral metrics can provide a 9 – 14% uplift. This is very promising, as trend and level as constructed here are very rudimentary attempts at time series models, and potentially greater uplift could be possible in conjunction with more sophisticated curve fitting models.

Table 4. Threshold 300 multivariate result.

	Dependent variable:			
	Disorder Period			
	(1)	(2)	(3)	(4)
Max Eigen Gap	175.36***		109.04***	
	(30.80)		(33.42)	
Mean Eigen Gap		2622.20***	1971.12***	
		(404.42)	(445.21)	
Level	-2.89***	-3.38***	-3.30***	-2.80***
	(0.45)	(0.45)	(0.44)	(0.47)
Trend	0.01***	0.02***	0.02***	0.01***
	(0.003)	(0.003)	(0.003)	(0.003)
Constant	732.30**	607.17***	578.38***	892.56***
	(42.30)	(53.90)	(53.78)	(33.20)
Observations	300	300	300	3200
R^2	0.20	0.22	0.25	0.11
Adjusted R^2	0.19	0.060.22	0.24	0.11

Note: * p<0.1; ** p<0.05; *** p<0.01.

Also promising, comparing across the models in Table 1 and Table 3, is the consistency of the signs of the coefficients under the different instantiations. The magnitudes, for Max and Mean Eigen gap variables, are, at least, relatable.

6 Discussion

In the context of sustainable development, the complex adaptive systems framework can help address the coupling of nature constraint and opportunity with population dynamics and individual agency.

Taking our previous research a step further with the introduction of spectral analysis of the social network fabric created by the game, we contribute both to the understanding of complex, strategically driven economic societies, as well as to the understanding of the value of theoretical physics methods in a new domain. The potential savings to computational costs in the pursuit of understanding complex games are potentially substantial and could represent a significant advance in the practice of artificial economics.

Furthermore, the above research gets a set of questions of fundamental interest to scientists empirically researching complex systems across a number of domains – given an unknown process that seems to be trending towards synchronization, how might we anticipate the likely period of frustration before such synchronization occurs? By utilizing our version of the HANDY model and asking this question at the same point in "time" in each simulation, with no foreknowledge of the eventual occurrence of synchronization, we realistically simulate this challenge and take a small step towards discovering useful information for this task from the spectral gap of the Laplacian of the dynamic connectivity matrix, and also towards forecasting even fuzzy definitions of synchronization in complex systems, where explicit characterization and analytical solution is infeasible.

Multiple outstanding and significant questions remain. For example, the consistency of coefficient signs across different initial parameter settings is of great interest. Even more trivial questions, such as the ability of spectral information to predict synchronization when extracted from dynamic connectivity matrices at different points in time, warrant further exploration. Obviously, the suitability of other variables characterizing the individual entities could be explored exhaustively, in the context of both intuition as well as the known, functional qualities of these variables as they describe the behavior of complex systems (but where their cyclical tendencies still resist analytical solution). Vectors of information from successive spectral analyses of evolving dynamic connectivity matrices, from one or possible more edges utilizing multiple variables, interactive terms using spectral information, and utilization of spectral information in non-linear models, such as neural networks, are all suitable subjects worthy of further research. These inquiries could be conducted on the basis of an underlying data generating process from any of the vast number of agent-based models emerging across various disciplines.

As always, one of the few significant obstacles to this line of research is computational expense; however, the flip side of such large parameter spaces is a very rich and seemingly never ending line of research avenues!

References

1. Abdollahian, M., Yang, Z., Coan, T., & Yesilada, B.: Human Development Dynamics: An Agent-Based Simulation of Macro Social Systems and Individual Heterogeneous Evolutionary Games. Complex Adaptive Systems Modeling, 1(1), 1-17 (2013)
2. Abdollahian, M., Yang, Z., & deWerk Neal, P.: Human Development Dynamics: An Agent-Based Simulation of Adaptive Heterogeneous Games and Social Systems. Social Computing, Behavioral-Cultural Modeling and Prediction, 3-10 (2014)
3. Abdollahian, M., Yang, Z., deWerk Neal, P., & Kaplan, J.: Human Development Dynamics: Network Emergence in an Agent-Based Simulation of Adaptive Heterogeneous Games and Social Systems. In Agent-Based Approaches in Economic and Social Complex Systems VIII (pp. 3-14). Springer Japan (2015)
4. Arenas, A., A. Diaz-Guilera, and C.J. Perez-Vicente. Synchronization Reveals Topological Scales in Complex Networks. Physical Review Letters, 96(11), 114102 (2006)
5. Banerjee, A., & Yakovenko, V. M.: Universal Patterns of Inequality. New Journal of Physics, 12(7), 075032 (2010)
6. Brander, J. A., & Taylor, M. S.: The Simple Economics of Easter Island: A Ricardo-Malthus model of renewable resource use. American Economic Review, 119-138 (1998)
7. Chua, A.: World on fire: How Exporting Free Market Democracy Breeds Ethnic Hatred and Global Instability. Anchor (2003)
8. Drăgulescu, A., & Yakovenko, V. M.: Exponential and Power-Law Probability Distributions of Wealth and Income in the United Kingdom and the United States. Physica A: Statistical Mechanics and its Applications, 299(1), 213-221 (2001)

9. Feng, Y., Kugler, J., & Zak, P. J.: The Politics of Fertility and Economic Development. International Studies Quarterly, 44(4), 667-693 (2000)
10. McKelvey, B.: Avoiding Complexity Catastrophe in Coevolutionary Pockets: Strategies for Rugged Landscapes. Organization Science, 10(3), 294-321 (1999)
11. Miller, J. H., & Page, S. E.: Complex Adaptive Systems: An Introduction to Computational Models of Social Life. Princeton university press (2009)
12. Motesharrei, S., Rivas, J., & Kalnay, E.: Human and Nature Dynamics (HANDY): Modeling Inequality and Use of Resources in the Collapse or Sustainability of Societies. Ecological Economics, 101, 90-102 (2014)
13. Neal, Patrick deWerk. Policy Implications of Econophysics: Oscillators, Coupling and Causality. (2016)
14. Nowak, M. A., & Sigmund, K.: Evolution of Indirect Reciprocity by Image Scoring. Nature, 393(6685), 573-577 (1998)
15. Redman, C. L., James, S. F. & Paul, D. R. J. (Eds.): The Archaeology of Global Change: The Impact of Humans on Their Environment. Smithsonian Books (2004)
16. Yang, Z.: An Agent-Based Dynamic Model of Politics, Fertility and Economic Development. Proceedings of The 20th World Multi-Conference on Systemics, Cybernetics and Informatics (2016)